

アルゴリズムとデータ構造入門

2.2.4 図形言語 (Picture Language)

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The Eighth Commandment

Use `help` function to abstract from representations.

The Ninth Commandment

Abstract common patterns with a new function.

The Tenth Commandment

Build functions to collect more than one value at a time.

(Friedman, et al. "The Little Schemer", MIT Press)

試験欠席者は
所定の手続きを

1



12月12日・本日のメニュー

今日は必修課題の説明です.

- 2.2.4 Picture Language (図形言語)
- Space Padding Functions
- Fractal (Self-Similarity)
 - Hilbert curve
 - Koch snowflake
 - Sierpinski's Gasket
 - Peano curve
- Square limit variation
解説: 和田英一「関数画家」, Webにあり.
- Circle limit

4

図形言語 Picture language とは



図形言語での学習目標

- 図形操作の抽象化
- 実際に図形言語で, フラクタル, 空間充填曲線が作成できる技術

6



図形言語の使い方 (README.tustk)

```
% tustk2
> (load '/usr/local/lib/tustk/demos/picl)
> (load '/usr/local/lib/tustk/demos/picl-test)
> (make-canvas frm1) ; 標準フレーム用キャンバスの作成
> ((square-limit wave 2) frm1)
> (tk:update) ; 上記実行で描画されなかった場合や
; 再描画が必要なときに実行
> (clear) ; 描いた絵を消す
> ((square-limit wave 4) frm1)
> (clear)
> ((squash-inwards wave) frm1)
> (forget-canvas) ; 現在のキャンバスを削除
> (make-canvas frm2) ; 傾いたフレーム用キャンバスの作成
> ((square-limit wave 4) frm2)
> (output-canvas 'wave.ps) ; 絵をPostScriptファイルへ出力
```



図形言語の使い方 (README.tustk)

- *bg-color* ; 背景色の定義
- *line-color* ; 描画線の定義
- (set! *bg-color* <色>) ; 色の設定を変更
- <色>; 色名 (blue, "red", ...), RGB指定 ("#RRGGBB")
- 図形は painter で定義
 - ・ 点 (make-vect <x-coordinate> <y-coordinate>)
 - ・ 線(セグメント) (make-segment <from-point> <to-point>)
 - ・ 線画 (segments->painter <list-of-segments>)
 - ・ 多角形(ポリゴン) (vectors->painter <list-of-points>)
 - ・ (vectors->painter <list-of-points> <smooth-or-not> <degree-of-smoothing> <filling-color>)
 - ・ (pgm-file->painter <file-name>) ; GIF/PPM/PGM

```
(define sicmp (pgm-file->painter
  "/usr/local/lib/tustk/demos/sicmp.ppm"))
((square-limit sicmp 4) frm1)
```



wave の定義

線を次々描いていく.

```
(define wave
  (segments->painter
    (list (make-segment (make-vect 0.25 0.00) (make-vect 0.35 0.50))
          (make-segment (make-vect 0.35 0.50) (make-vect 0.30 0.55))
          (make-segment (make-vect 0.30 0.55) (make-vect 0.20 0.45))
          (make-segment (make-vect 0.20 0.45) (make-vect 0.25 0.00))
          (make-segment (make-vect 0.00 0.80) (make-vect 0.20 0.55))
          (make-segment (make-vect 0.20 0.55) (make-vect 0.30 0.60))
          (make-segment (make-vect 0.30 0.60) (make-vect 0.40 0.60))
          (make-segment (make-vect 0.40 0.60) (make-vect 0.35 0.80))
          (make-segment (make-vect 0.35 0.80) (make-vect 0.40 1.00))
          (make-segment (make-vect 0.60 1.00) (make-vect 0.65 0.80))
          (make-segment (make-vect 0.65 0.80) (make-vect 0.60 0.60))
          (make-segment (make-vect 0.60 0.60) (make-vect 0.70 0.60))
          (make-segment (make-vect 0.70 0.60) (make-vect 1.00 0.40))
          (make-segment (make-vect 1.00 0.20) (make-vect 0.65 0.50))
          (make-segment (make-vect 0.65 0.50) (make-vect 0.75 0.00))
          (make-segment (make-vect 0.60 0.00) (make-vect 0.50 0.20))
          (make-segment (make-vect 0.50 0.20) (make-vect 0.40 0.00)))))
```



wave の変形

```

(define red-wave
  (vecs->painter
    (list
      (make-vect 0.25 0.00) (make-vect 0.35 0.50)
      (make-vect 0.30 0.55) (make-vect 0.20 0.45)
      (make-vect 0.00 0.60) (make-vect 0.00 0.80)
      (make-vect 0.20 0.55) (make-vect 0.30 0.60)
      (make-vect 0.40 0.60) (make-vect 0.35 0.80)
      (make-vect 0.40 1.00) (make-vect 0.60 1.00)
      (make-vect 0.65 0.80) (make-vect 0.60 0.60)
      (make-vect 0.70 0.60) (make-vect 1.00 0.40)
      (make-vect 1.00 0.20) (make-vect 0.65 0.50)
      (make-vect 0.75 0.00) (make-vect 0.60 0.00)
      (make-vect 0.50 0.20) (make-vect 0.40 0.00) )
    #f 0 'red ))

```




点をつないで、ポリゴンを作成し、色を塗る.

10



lambda の定義

```

(define red-letterlambda
  (vecs->painter
    (list
      (make-vect .45 .60) (make-vect .25 .20)
      (make-vect .25 .20) (make-vect .20 .20)
      (make-vect .20 .20) (make-vect .20 .10)
      (make-vect .20 .10) (make-vect .30 .10)
      (make-vect .30 .10) (make-vect .50 .50)
      (make-vect .50 .50) (make-vect .70 .10)
      (make-vect .70 .10) (make-vect .80 .10)
      (make-vect .80 .10) (make-vect .80 .20)
      (make-vect .80 .20) (make-vect .75 .20)
      (make-vect .75 .20) (make-vect .40 .90)
      (make-vect .40 .90) (make-vect .30 .90)
      (make-vect .30 .90) (make-vect .30 .80)
      (make-vect .30 .80) (make-vect .35 .80)
      (make-vect .35 .80) (make-vect .45 .60) )
    #f 0 'red ))

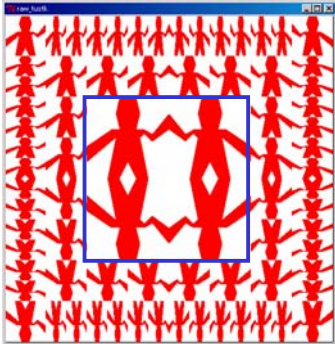
```



12



square-limit n



これから square-limit を作る.
左はレベル2
まず、中央

14



flipped-pair (拡張版)

```

(define wave2
  (beside wave (flip-vert wave)))
(define wave4 (below wave2 wave2))

(define (flipped-pairs painter)
  (let ((painter2
        (beside painter
          (flip-vert painter) )))
    (below painter2 painter2)))

```

こうすると

```

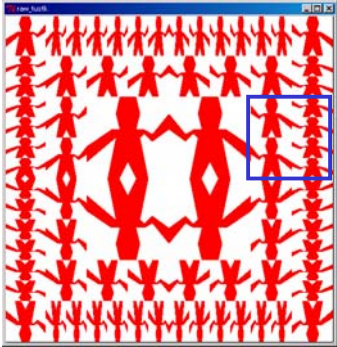
(define wave4
  (flipped-pairs wave))

```






square-limit *n*



これから
square-limit
を作る。
左はレベル2
次は、右側

16



right-split *n*

```

(define (right-split painter n)
  (if (= n 0)
      painter
      (let ((smaller
            (right-split painter (- n 1))))
        (beside painter
          (below smaller smaller) ))))

```



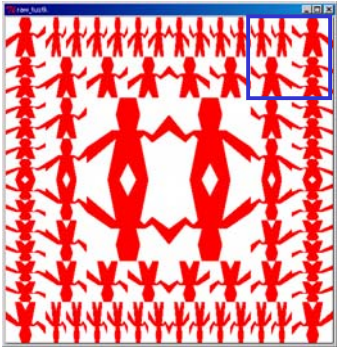
identity	right-split n-1
	right-split n-1

未定義の手続き
 (below bottom top)
 (beside left right)

17



square-limit n



これから square-limit を作る.
左はレベル2
今度は, 角

20



corner-split n

```

(define (corner-split painter n)
  (if (= n 0)
      painter
      (let ((up (up-split painter (- n 1)))
            (right (right-split painter (- n 1))) )
          (let ((top-left (beside up up))
                (bottom-right (below right right))
                (corner (corner-split painter (- n 1))) )
              (beside (below painter top-left)
                      (below bottom-right corner) )))))

```

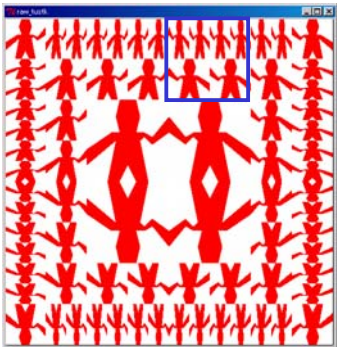
up-split $n-1$	up-split $n-1$	corner-split $n-1$
identity		right-split $n-1$
		right-split $n-1$

未定義の手続き
(below bottom top)
(beside left right)

21



square-limit n



これから square-limit を作る.
左はレベル2
最後は, 上

24



Ex.2.44 up-split

```

(define (up-split painter n)
  (if (= n 0)
      painter
      (let ((smaller
              (up-split painter (- n 1))))
          (below painter
                  (beside smaller smaller) )))))

```



up-split n-1	up-split n-1
identity	

未定義の手続き
 (below bottom top)
 (beside left right)

25

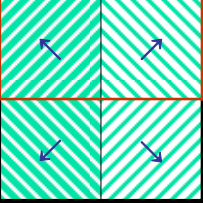


square-limit *n*

```

(define (square-limit painter n)
  (let ((quarter
          (corner-split painter n)))
    (let ((half
            (beside
              (flip-horiz quarter)
              quarter)))
      (below (flip-vert half)
              half))))

```







square-limit *n*



これで
square-limit
が完成.
左はレベル1

28



split群を抽象化

```

(define right-split (split beside below))
(define up-split (split below beside))
(define (split op1 op2)
  (op1 half (op2 quarter quarter)))

```

identity	right-split n-1
	right-split n-1

up-split n-1	up-split n-1
identity	



Escher's square-limit



和田:関数画家、『情報処理』、
Vol.46, No.10 (2005) 1163-1171
<http://www.ecs.soton.ac.uk/~ph/papers/funcgeo2.pdf>



Higher-order operations

```

(define (square-of-four tl tr bl br)
  (lambda (painter)
    (let ((top (beside (tl painter)
                        (tr painter)))
          (bottom (beside (bl painter)
                          (br painter))))
      (below bottom top))))

```

tl	tr
bl	br

未定義の手続き
 (below bottom top)
 (beside left right)



flipped-pairsの別の定義

```

(define (flipped-pairs painter)
  (let ((combine4
        (square-of-four
         identity flip-vert
         identity flip-vert )))
    (combine4 painter) ))

```

ident ity	flip- vert
ident ity	flip- vert

未定義の手続き
 (below bottom top)
 (beside left right)

35

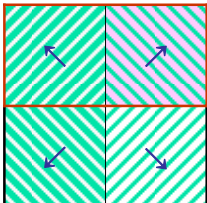


square-limit の別の定義

```

(define (square-limit painter n)
  (let ((combine4
        (square-of-four
         flip-horiz identity
         rotate180 flip-vert )))
    (combine4
     (corner-split painter n) )) )

```



flip- horiz	ident ity
rotat e180	flip- vert

36



12月12日・本日のメニュー

今日は必修課題の説明です.

- 2.2.4 Picture Language (図形言語)
具体的な描画手続きを定義
- Space Padding Functions
- Fractal (Self-Similarity)
 - Hilbert curve
 - Koch snowflake
 - Sierpinski's Gasket
 - Peano curve
- Square limit variation
解説: 和田英一「関数画家」, Webにあり.
- Circle limit

37

Frame coordinate map

- coordinate: unit squareに作成
- frameに写像

$$\text{Origin}(\text{Frame}) + x \cdot \text{Edge}_1(\text{Frame}) + y \cdot \text{Edge}_2(\text{Frame})$$

38

Frames

```
(define (frame-coord-map frame)
  (lambda (v)
    (add-vect
      (origin-frame frame)
      (add-vect (scale-vect (xcor-vect v)
                           (edge1-frame frame))
                (scale-vect (ycor-vect v)
                           (edge2-frame frame))
            )
    )))
```

```
((frame-coord-map a-frame)
 (make-vect 0 0) )
```

の返す値: (origin-frame a-frame)

39

Frames

```
(define (make-frame origin edge1 edge2)
  (list origin edge1 edge2) )
```

```
(define (make-frame origin edge1 edge2)
  (cons origin (cons edge1 edge2)) )
```

それぞれに対する選択子を書け。

40

Painters

```
(define (segments->painter segment-list)
  (lambda (frame)
    (for-each
      (lambda (segment)
        (draw-line
          ((frame-coord-map frame)
           (start-segment segment))
          ((frame-coord-map frame)
           (end-segment segment)))))
      segment-list)))
```

(foreach <procedure> <list-of-items>)

42

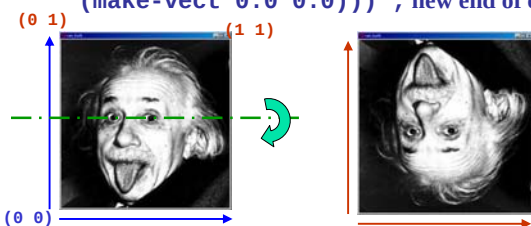
Transforming and combining painters

```
(define (transform-painter painter
  origin corner1 corner2)
  (lambda (frame)
    (let ((m (frame-coord-map frame)))
      (let ((new-origin (m origin)))
        (painter
          (make-frame new-origin
            (sub-vect (m corner1)
                      new-origin)
            (sub-vect (m corner2)
                      new-origin)))))))
```

43

Transforming and combining painters

```
(define (flip-vert painter)
  (transform-painter
   painter
   (make-vect 0.0 1.0) ; new origin
   (make-vect 1.0 1.0) ; new end of edge1
   (make-vect 0.0 0.0)) ; new end of edge2
```



44

Transforming and combining painters

```

(define (shrink-to-upper-right painter)
  (transform-painter
   painter
   (make-vect 0.5 0.5)
   (make-vect 1.0 0.5)
   (make-vect 0.5 1.0)))

```

Transforming and combining painters

```

(define (rotate90 painter)
  (transform-painter
   painter
   (make-vect 1.0 0.0)
   (make-vect 1.0 1.0)
   (make-vect 0.0 0.0)))

```

Transforming and combining painters

```

(define (squash-inwards painter)
  (transform-painter
   painter
   (make-vect 0.0 0.0)
   (make-vect 0.65 0.35)
   (make-vect 0.35 0.65)))

```

傾いたフレーム

```
(define frm2
  (make-frame
    (make-vect 0 0)
    (make-vect 1.0 0)
    (make-vect 0.5 0.5) )
```



48

beside

```
(define (beside painter1 painter2)
  (let ((split-point (make-vect 0.5 0.0)))
    (let ((paint-left
            (transform-painter
              painter1
              (make-vect 0.0 0.0)
              split-point
              (make-vect 0.0 1.0)))
          (paint-right
            (transform-painter
              painter2
              split-point
              (make-vect 1.0 0.0)
              (make-vect 0.5 1.0))))
      (lambda (frame)
        (paint-left frame)
        (paint-right frame)))))
```

49

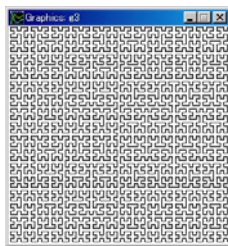
above m:n で分割

```
(define (above painter1 painter2 . l)
  (let* ((m (if (null? l) 1 (car l)))
        (n (if (or (null? l) (null? (cdr l)))
                1 (cadr l)))
        (r (/ n (+ m n))))
    (split-point (make-vect 0.0 r))
    (let ((paint-lower
            (transform-painter painter2
              (make-vect 0.0 0.0)
              (make-vect 1.0 0.0)
              split-point))
          (paint-upper
            (transform-painter painter1
              split-point
              (make-vect 1.0 r)
              (make-vect 0.0 1.0))))
      (lambda (frame)
        (paint-lower frame)
        (paint-upper frame)))))
```

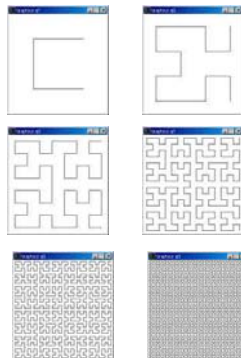
50



Hilbert curve の作成方法



(hilbert 5)



56



Hilbert curve の作成方法

4つの基本形

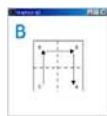
1. 基本形A: $D \Rightarrow A \Rightarrow A \Rightarrow B$
2. 基本形B: $C \Rightarrow B \Rightarrow B \Rightarrow A$
3. 基本形C: $B \Rightarrow C \Rightarrow C \Rightarrow D$
4. 基本形D: $A \Rightarrow D \Rightarrow D \Rightarrow C$

基本形A



分解形A

基本形B



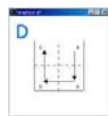
分解形B

基本形C



分解形C

基本形D



分解形D

58



Hilbert curve の手続き

1. 各基本形に対して、レベル0ならコ型を書くための頂点のリストを求める。
2. さもないと、分解形を再帰的に呼び出し、頂点を求める。
3. 求まった頂点リストから segment を求め painter を `vectors->segment` と `segments->painter` を使って作成する。

```
(vectors->segment <list of vectors>)
(segments->painter <list of segments>)
```

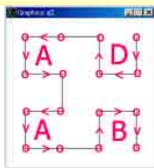
59



Hilbert curve の手続き

```
(define (hilbert-a x0 y0 x1 y1 i)
  (let ((xs (/ (+ (* 3.0 x0) x1) 4.0))
        (ys (/ (+ (* 3.0 y0) y1) 4.0))
        (xm (/ (+ x0 x1) 2.0))
        (ym (/ (+ y0 y1) 2.0))
        (xl (/ (+ x0 (* 3.0 x1)) 4.0))
        (yl (/ (+ y0 (* 3.0 y1)) 4.0)) )
    (if (= i 0)
        (list (make-vect xl yl) (make-vect xs yl)
              (make-vect xs ys) (make-vect xl ys) )
        (append (hilbert-d xm ym x1 y1 (- i 1))
                  (hilbert-a x0 ym xm y1 (- i 1))
                  (hilbert-a x0 y0 xm ym (- i 1))
                  (hilbert-b xm y0 x1 ym (- i 1)) )))))

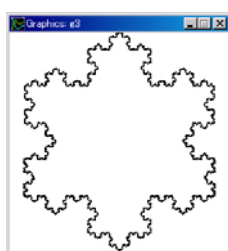
(define (hilbert n)
  (segments->painter
   (vectors->segments (hilbert-a 0.0 0.0 1.0 1.0 n)))) )
```



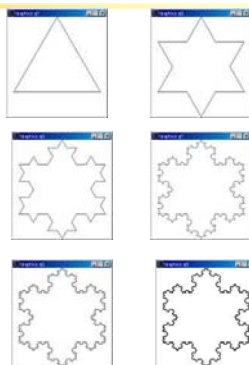
60



Koch snowflake の作成方法



(koch 5)

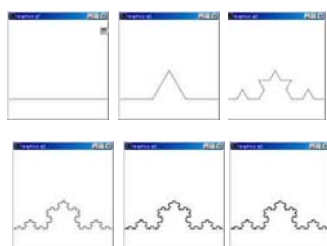
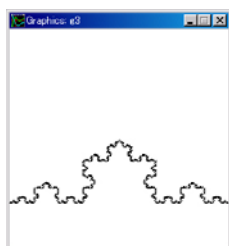


61



Koch snowflake の作成方法

線分の分解



63



Koch snowflake の手続き

1. 各線分に対して、レベル0なら、三角形の頂点リストを求める。
2. さもなければ、分解形を再帰的に呼び出し、頂点を求める。
3. 求まった頂点リストから segment を求め painter を vectors->segment と segments->painterを使って作成する。

```
(vectors->segment <list of vectors>)
(segments->painter <list of segments>)
```

64



Koch snowflake の手続き(続)

```
(define (koch-line x0 y0 x1 y1 r i)
  (if (= i 0)
      (list (make-vect x0 y0) (make-vect x1 y1))
      (let* ((r1 (/ r 3.0))
              (x3 (/ (- x1 x0) 3.0))
              (y3 (/ (- y1 y0) 3.0))
              (xs (/ (+ (* 2.0 x0) x1) 3.0))
              (ys (/ (+ (* 2.0 y0) y1) 3.0))
              (xl (/ (+ x0 (* 2.0 x1)) 3.0))
              (yl (/ (+ y0 (* 2.0 y1)) 3.0))
              (xm (+ (* 0.5 x3) (* 0.866 y3) xs))
              (ym (+ (* 0.5 y3) (* -0.866 x3) ys)))
        (append (koch-line x0 y0 xs ys r1 (- i 1))
                  (koch-line xs ys xm ym r1 (- i 1))
                  (koch-line xm ym xl yl r1 (- i 1))
                  (koch-line xl yl x1 y1 r1 (- i 1))
                ))))
```



65



Koch snowflake の手続き(続)

```
(define (koch n)
  (let* ((h (/ 0.75 0.86))
         (x0 (/ (- 1.0 h) 2))
         (x1 (- 1.0 x0)))
    (segments->painter
     (vectors->segments
      (append
       (koch-line x0 0.25 x1 0.25 1 n)
       (koch-line x1 0.25 0.5 1.0 1 n)
       (koch-line 0.5 1.0 x0 0.25 1 n)
      )))))
```



let* は let と違い、変数値対を順番に評価

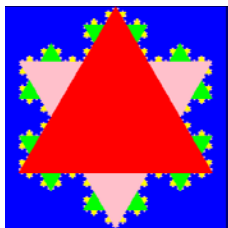
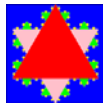
66



色つきKoch curve の手続き

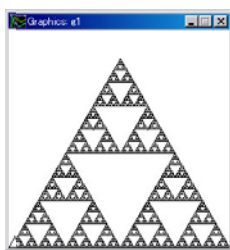
```
(define (koch-fill n . args)
  (let* ((h (/ 0.75 0.86))
        (x0 (/ (- 1.0 h) 2))
        (x1 (- 1.0 x0))
        (color (if (null? args) 'red (car args)))) )
    (vectors->painter
      (append (koch-line x0 0.25 x1 0.25 1 n)
              (koch-line x1 0.25 0.5 1.0 1 n)
              (koch-line 0.5 1.0 x0 0.25 1 n))
      #f 0 color )))

(define (fun-koch x)
  ((koch-fill 5 'pink) x)
  ((koch-fill 4 'white) x)
  ((koch-fill 3 'yellow) x)
  ((koch-fill 2 'green) x)
  ((koch-fill 1 'pink) x)
  ((koch-fill 0 'red) x)
)
```

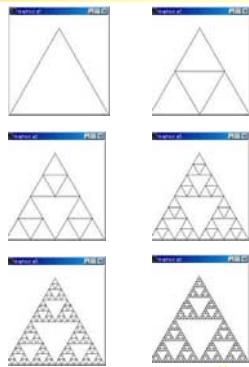




Sierpinski's Gasket の作成方法



(sierpiski_6)



68



Sierpinski's Gasket の手続き

```
(define (gasket x0 y0 x1 y1 i)
  (let* ((xm (/ (+ x0 x1) 2.0))
        (ym (+ (* (- x1 x0) 0.866) y0))
        (xs (/ (+ (* 3.0 x0) x1) 4.0))
        (xl (/ (+ (* 3.0 x1) x0) 4.0))
        (ys (+ (* (- x1 x0) 0.433) y0)) )
    (if (= i 0)
        (list (make-vect x0 y0) (make-vect x1 y1)
              (make-vect (/ (+ x0 x1) 2.0) ym)
              (make-vect x0 y0) )
        (append (gasket x0 y0 xm y0 (- i 1))
                (gasket xm y0 x1 y0 (- i 1))
                (list (make-vect x0 y0))
                (gasket xs ys xl ys (- i 1))
                (list (make-vect x0 y0) )))))

(define (sierpinski n)
  (segments->painter
    (vectors->segments (gasket 0.0 0.0 1.0 0.0 n)) ))
```



69



色つきSierpinski's Gasket 手続き

```
(define (gasket-fill x0 y0 x1 y1 i color)
  (let* ((xm (/ (+ x0 x1) 2.0))
        (ym (+ (* (- x1 x0) 0.866) y0))
        (xs (/ (+ (* 3.0 x0) x1) 4.0))
        (xl (/ (+ (* 3.0 x1) x0) 4.0))
        (ys (+ (* (- x1 x0) 0.433) y0)) )
    (if (= i 0)
        (list (vects->painter
              (list (make-vect x0 y0)
                    (make-vect x1 y1)
                    (make-vect xm ym) )
              #f 0 color))
        (append (gasket-fill x0 y0 xm y0 (- i 1) color)
                (gasket-fill xm y0 x1 y0 (- i 1) color)
                (gasket-fill xs ys xl ys (- i 1) color) ))))

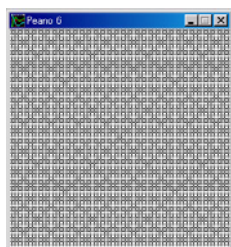
(define (sierpinski-fill n . args)
  (let ((color (if (null? args) 'red (car args))))
    (do ((i (gasket-fill 0.0 0.0 1.0 0.0 n color) (cdr i)))
        ((null? i))
        (i frm1) )))
```



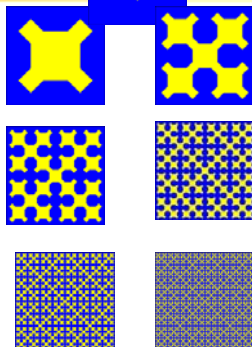
70



Peano Curve の作法



(peano 6)



71

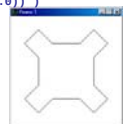
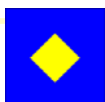


Peano Curve の手続き

```
(define (peano-a x0 y0 x1 y1 i)
  (append (peano-a-1 x0 y0 x1 y1 i)
          (peano-a-2 x0 y0 x1 y1 i) ))

(define (peano-a-1 x0 y0 x1 y1 i)
  (let ((xs (/ (+ (* 3.0 x0) x1) 4.0)) (ys (/ (+ (* 3.0 y0) y1) 4.0))
        (xm (/ (+ x0 x1) 2.0)) (ym (/ (+ y0 y1) 2.0))
        (xl (/ (+ x0 (* 3.0 x1)) 4.0)) (yl (/ (+ y0 (* 3.0 y1)) 4.0)) )
    (if (= i 0)
        (list (make-vect xm yl) (make-vect xs ym))
        (append (peano-a-1 xm ym x1 y1 (- i 1))
                (peano-d-1 x0 ym xm y1 (- i 1))
                (peano-d-2 x0 ym xm y1 (- i 1))
                (peano-a-1 x0 y0 xm ym (- i 1)) ))))

(define (peano-a-2 x0 y0 x1 y1 i)
  (let ((xs (/ (+ (* 3.0 x0) x1) 4.0)) (ys (/ (+ (* 3.0 y0) y1) 4.0))
        (xm (/ (+ x0 x1) 2.0)) (ym (/ (+ y0 y1) 2.0))
        (xl (/ (+ x0 (* 3.0 x1)) 4.0)) (yl (/ (+ y0 (* 3.0 y1)) 4.0)) )
    (if (= i 0)
        (list (make-vect xm ys) (make-vect xl ym))
        (append (peano-a-2 x0 y0 xm ym (- i 1))
                (peano-b-1 xm y0 x1 ym (- i 1))
                (peano-b-2 xm y0 x1 ym (- i 1))
                (peano-a-2 xm ym x1 y1 (- i 1)) ))))
```



72



必修課題2: 2月15日午後5時締切

1. 気の利いたpainterを1種類作れ.
2. それをsquare-limit等に適用.
3. 空間充填曲線を1種類作れ.
(Hilbert curve, Peano curve, ...)
4. フラクタルを1種類作れ
(Koch Snowflake, Sierpinsky's Gasket, ...)

- レポートは紙, またはファイルで.
- プログラムはメールで okuno@i.kyoto-u.ac.jp
- 例は: <http://winnie.kuis.kyoto-u.ac.jp/>
- 教えてもらった場合には, 明記すること.
- 人と同じプログラム・作品(年度不問)は再提出.



73



随意課題3: 3月15日午後5時締切

circle-limit を作成せよ。

プログラムはメールで
okuno@i.kyoto-u.ac.jp









宿題: 12月18日午後5時締切

宿題は、次の計4問:

- Ex. 2.37, 2.38, 2.39, 2.42
- Ex.2.44~52は必修課題2-2に包含
- TAのレポート講評を見て復習すること.
- 教えてもらったときには明記すること.
- 人のレポートのコピーは厳禁(減点)



68
